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Two Illustrations of the Quantity Theory of Money: Breakdowns and Revivals

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By extending his data, we document the instability of low-frequency regression coefficients that Lucas (1980) used to express the quantity theory of money. We impute the differences in these regression coefficients to differences in monetary policies across periods. A DSGE model estimated over a subsample like Lucas's implies values of the regression coefficients that confirm Lucas's results for his sample period. But perturbing monetary policy rule parameters away from the values estimated over Lucas's subsample alters the regression coefficients in ways that reproduce their instability over our longer sample. (*JEL* C51, E23, E31, E43, E51, E52)

Robert E. Lucas Jr. (1980) used near unit slopes of univariate regressions of moving averages of inflation and interest rates on money growth for the United States for the period 1953–1977 to illustrate "two central implications of the quantity theory of money: that a given change in the rate of change in the quantity of money induces (i) an equal change in the rate of price inflation; and (ii) an equal change in nominal rates of interest.'' Lucas said that those two quantity-theoretic propositions

> …possess a combination of theoretical coherence and empirical verification shared by no other propositions in monetary economics. By "theoretical coherence,'' I mean that each of these laws appears as a characteristic of solutions to explicit theoretical models of idealized economies, models which give some guidance as to why one might expect them to obtain in reality, *also as to conditions under which one might expect them to break down* (emphasis added) (1980, p. 1005).

This paper extends Lucas's analysis to a longer US dataset and uses an explicit theoretical model to identify conditions on monetary policy that cause the unit slopes to "obtain in reality'' as well as to "break down.'' We find that Lucas's low-frequency

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regression slopes are not stable over time, an empirical outcome that we explain in terms of quantitative versions of our theoretical "break down" conditions. In our theoretical model, the regression coefficients on moving averages depend on monetary policy.[1] By freezing all nonmonetary policy structural parameters at values estimated over a sample period approximating Lucas's, we display variations in two parameters of a monetary policy rule that push the population values of those low frequency slopes over a range that covers the empirical outcomes found in our extended sample. In this way, we construct different monetary policy rules that, in the context of our structural model, can explain the differences over time in the estimated low-frequency regression slopes.[2]

Why have we written this paper now? For most of the last 25 years, the quantity theory of money has been sleeping, but during the last year, unprecedented growth in leading central banks' balance sheets has prompted some of us to worry because the quantity theory has slept before, only to reawaken. Our DSGE model tells us that what puts those quantity-theoretic unit slopes to sleep is a monetary policy rule that responds to inflationary pressure aggressively enough to prevent the emergence of persistent movements in money growth, and that what awakens them is a monetary policy rule that accedes to persistent movements in money growth by responding too weakly to inflationary pressure. It seems timely to characterize the features of monetary policy rules needed to arrest the reemergence of the empirical patterns that Lucas takes as tell-tale signs of the quantity theory.

To set the stage for our empirical findings, Section I recounts Charles H. Whiteman's (1984) observation that the slope of Lucas's scatter plot estimates the sum of coefficients in a long two-sided distributed lag regression, then indicates how the population value of that slope is linked to the parameters of a state space representation for either a vector autoregression or a DSGE model. Section II reports scatter plots and sums of distributed lag coefficients constructed from estimates of both time-invariant and time-varying vector autoregressions. These document substantial instability of Lucas's two scatter plot slopes. Section III uses Bayesian methods to estimate our DSGE model over a subperiod approximating Lucas's, verifies that the estimated structural parameters confirm Lucas's unit slope findings over his sample, and then, by perturbing monetary policy while freezing other model parameters, indicates how variations in the conduct of monetary policy cause outcomes to break down in ways that can account for the

[1] Lucas interpreted his unit slope findings as measuring "…the extent to which the inflation and interest rate experience *of the postwar period* can be understood in terms of purely classical, monetary forces" (italics added). Lucas's purpose, including the qualification we have italicized, was precisely to indicate that the unit slope finding depends for its survival on maintenance of the monetary policy in place during the 1953–1977 period.

[2] Why, among the list of possible structural parameters in our model, do we confine ourselves to the monetary policy rule when searching for the cause of observed instability in the two low-frequency regressions? We have carried out robustness exercises (e.g., perturbed values for nonmonetary policy rule parameters within the structural model presented in the text and even experiments within a calibrated version of a quite different structural model of Lucas 1975), and these have pushed us toward emphasizing the monetary policy rule as the most likely cause of the low-frequency regression coefficient instabilities that we are trying to explain. Furthermore, DSGE models like the one we are using were intentionally designed as devices to use the cross-equation restrictions emerging from rational expectations models in the manner advocated by Lucas (1972) and Sargent (1971), to interpret how regressions involving inflation would depend on monetary and fiscal policy rules. We think that we are using our structural model in one of the ways its designers intended.

observed range of instability in the slopes of the scatter plots. Section IV offers concluding remarks.

I. Lucas's and Whiteman's Methods

For US data over 1955–1975, Lucas (1980) plotted moving averages of inflation and a nominal interest rate on the y axis against the same moving average of money growth on the x axis. In this section, we revisit Whiteman's (1984) argument that the slope of the regression through the scatter plot of Lucas's moving averages can be approximated as the sum of distributed lag coefficients, and that this sum can be computed using the spectral density implied by a state space representation of the data.

A. *The Slope of Scatter Plots of Filtered Series*

For a scalar series x_t and $\beta \in [0,1)$, Lucas (1980) constructed moving averages $\overline{x}_t(\beta) = \alpha \sum_{k=-n}^{n} \beta^{|k|} x_{t+k}$ where choosing α according to $\alpha = (1-\beta)^2/(1-\beta^2-2\beta^{n+1}(1-\beta))$ made the sum of weights equal one.

Whiteman (1984) observed that fitting straight lines through scatter plots of moving averages is an informal way of computing sums of coefficients in long two-sided distributed lag regressions. Let $\{y_t, z_t\}$ be a bivariate jointly covariance stationary process with unconditional means of zero and consider the two-sided infinite least-squares projection of y_t on past, present, and future z's:

$$(1) \qquad y_t = \sum_{j=-\infty}^{\infty} h_j z_{t-j} + \epsilon_t,$$

where ϵ_t is a random process that satisfies the population orthogonality conditions $E\epsilon_t z_{t-j} = 0 \ \forall_j$. Let the spectral densities of y and z be denoted $S_y(\omega)$ and $S_z(\omega)$, respectively, and let the cross-spectral density be denoted $S_{yz}(\omega)$. Let the Fourier transform of $\{h_j\}$ be $\tilde{h}(\omega) = \sum_{j=-\infty}^{\infty} h_j e^{-i\omega j}$. Then

$$(2) \qquad \tilde{h}(\omega) = \frac{S_{yz}(\omega)}{S_z(\omega)}$$

and the sum of the distributed lag regression coefficients is

$$(3) \qquad \sum_{j=-\infty}^{\infty} h_j = \tilde{h}(0) = \frac{S_{yz}(0)}{S_z(0)}.$$

Whiteman (1984) showed that for β close to 1, the regression coefficient b_f of a Lucas moving average $\overline{y}_t(\beta)$ on a Lucas moving average $\overline{x}_t(\beta)$ satisfies

$$(4) \qquad b_f \approx \frac{S_{yz}(0)}{S_z(0)} = \tilde{h}(0).$$

B. *Mappings from VAR and DSGE Models to* $\tilde{h}(0)$

Time-invariant versions of our VARs and of our log-linear DSGE models can both be represented in terms of the state space system

$$X_{t+1} = AX_t + BW_{t+1}, \qquad (5)$$

$$Y_{t+1} = CX_t + DW_{t+1},$$

where X_t is an $n_X \times 1$ state vector, W_{t+1} is an $n_W \times 1$ Gaussian random vector with mean zero and unit covariance matrix and that is distributed identically and independently across time, Y_t is an $n_Y \times 1$ vector of observables, and A, B, C, D are matrices, with the eigenvalues of A being bounded strictly above by unity (A can be said to be a "stable" matrix). DSGE models make elements of the matrices A, B, C, D be (nonlinear) functions of a vector of structural parameters η, some of which describe monetary policy.

The spectral density matrix of Y is[3]

$$S_Y(\omega) = C(I - Ae^{-i\omega})^{-1}BB'(I - A'^{i\omega})^{-1}C' + DD'. \qquad (7)$$

The Fourier transform of the population regression coefficients $\tilde{h}(\omega)$ can be computed from formula (2) where $S_{yz}(\omega)$, the cross spectrum between y and z, and $S_z(\omega)$, the spectrum of z, are the appropriate elements of $S_Y(\omega)$. In figures 6 and 7 in Section III, we summarize the mapping to $\tilde{h}(0)$ from the elements of the parameter vector η that govern monetary policy.

II. Scatter Plots and Regressions

In this section, we present data and extend Lucas's scatter plots of moving averages of money growth and inflation as well as money growth and the nominal interest rate. Then we compute regressions on filtered data and sums of distributed lag coefficients by applying "temporary" versions of formulas (2) and (7) to a VAR with drifting coefficients and stochastic volatility. Both the scatter plots and the regressions point to instability in the two low-frequency relationships that Lucas took to signify the quantity theory.[4]

A. *Data*

We use quarterly US data. Real and nominal GDP (M2 stock) are available from the FRED database since 1947:I (1959:I). Prior to that, we apply backward the

[3] The spectral density matrix is the Fourier transform of the sequence of autocovariance matrices $EY_tY'_{t-j}$, $j = -\infty, \ldots, -1, 0, 1, \ldots, +\infty$ whose typical element can be recovered from $S_Y(\omega)$ via the inversion formula

$$EY_tY'_{t-j} = \left(\frac{1}{2}\pi\right)\int_{-\pi}^{\pi} S_Y(\omega)\, e^{i\omega j}\, d\omega. \qquad (6)$$

[4] Rather than estimating $\tilde{h}(1)$ by first estimating a VAR as we do, another worthwhile strategy would be to apply the dynamic ordinary least squares or the dynamic generalized least squares estimator of James H. Stock and Mark W. Watson (1993) to estimate $\tilde{h}(1)$ as the simple regression coefficient of $\bar{y}_t$ on $\bar{z}_t$. Procedures of Peter C.B. Phillips (1991) can also be applied to estimate $\tilde{h}(1)$ viewed as a regression coefficient.

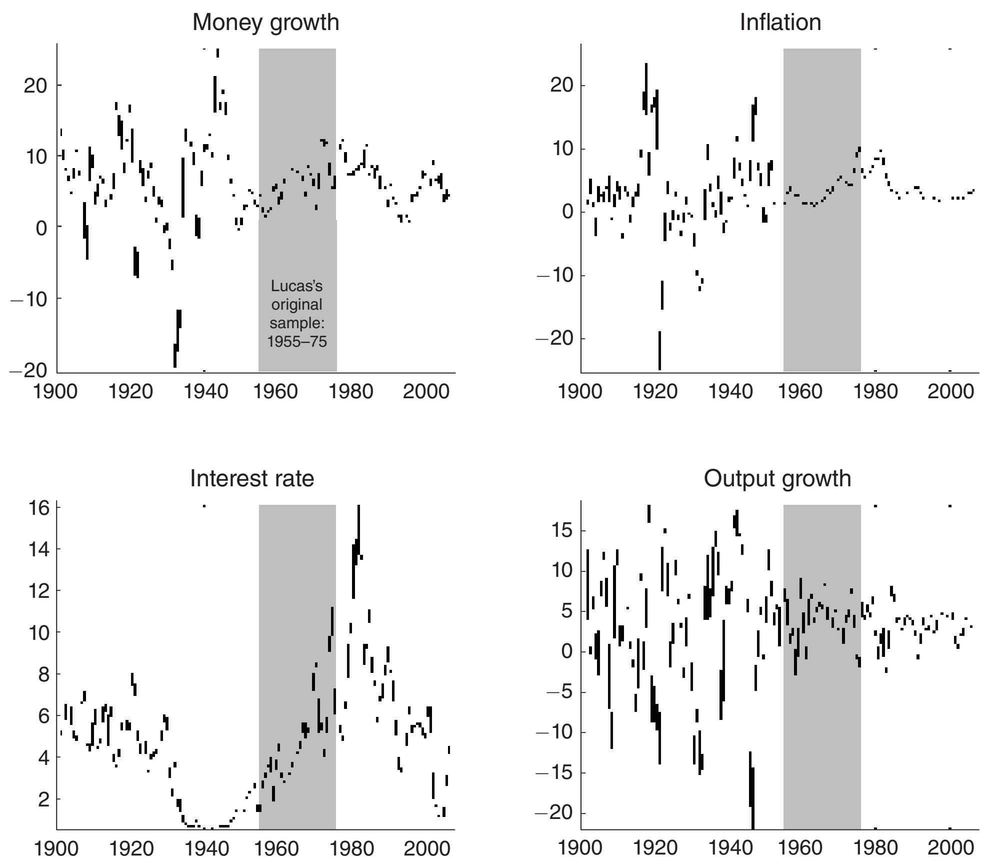

FIGURE 1. M2 GROWTH, GNP/GDP DEFLATOR INFLATION, 6-MONTH COMMERCIAL PAPER RATE, AND REAL GNP/GDP GROWTH.
(*Sample*: 1900:I–2005:IV)

growth rates on the real GNP and M2 series constructed by Nathan S. Balke and Robert J. Gordon (1986).[5] As for the nominal short-term interest rate, we use the six-month commercial paper rate available from Balke and Gordon (1986) until 1983 and from the FRED database afterwards. Figure 1 displays year-on-year first differences of logs of raw variables. The interest rate is displayed in level. Figure 2 reports moving averages of the raw data using Lucas's $\beta = 0.95$ filter. The shaded regions in these two filters isolate the 1955–1975 period that Lucas focused on.

These figures reveal some striking patterns.

- Figure 1 reveals that for money growth, inflation, and output growth, but not for the interest rate, volatility decreased markedly after 1950.
- The filtered data in Figure 2 indicate that the shaded period studied by Lucas exhibits persistent increases in money growth, inflation, and the interest rate. These features let Lucas's two quantity-theoretic propositions leap off the page. However,
- For the filtered data, the shaded Lucas sample observations are atypical.

[5] As for M2, Balke and Gordon (1986) build upon Milton Friedman and Anna J. Schwartz (1963).

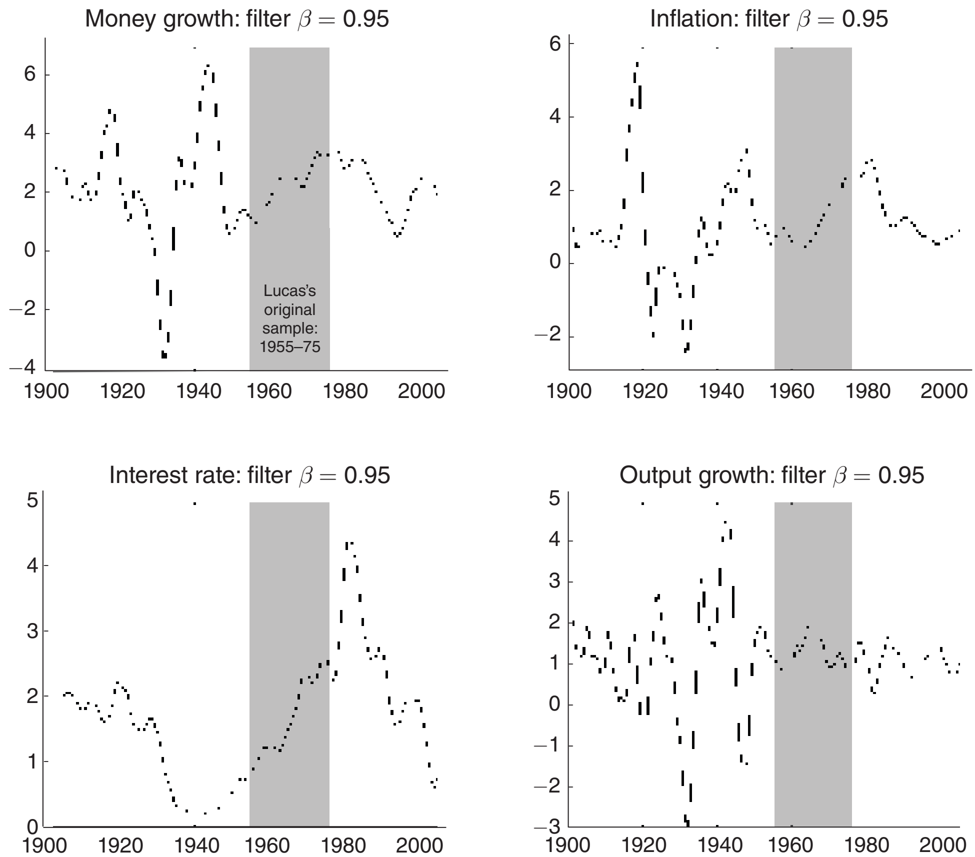

Figure 2. $\beta = 0.95$-Filtered Series for M2 Growth, GNP/GDP Deflator Inflation, 6-Month Commercial Paper Rate, and Real GNP/GDP Growth

B. *Scatter Plots*

Figures 3 and 4 show scatter plots of second quarter observations of each year of filtered series over selected subperiods in the sample 1900–2005. We selected the subsamples to include Lucas's subperiod, 1955–1975. In addition, we follow John F. Boschen and Christopher M. Otrok's (1994) comment on Mark E. Fisher and John J. Seater (1993) and split the sample around the Great Depression. To emphasize the link between Lucas's calculations and monetary policy regimes, we also present results for the periods 1960–1983 and 1984–2005, which are typically the focus of the literature on the great moderation. Altogether, we display six subperiods in different panels in figures 3 and 4. In subsection D, we show that the subsample instabilities presented in this section do not depend on the particular sample selection used here.

These graphs reveal the following patterns to us. The scatters of points can be said to align with the two quantity propositions in the 1955–75 and 1960–83 subperiods, and to a lesser extent between 1976 and 2005: the points adhere to lines that at least seem to be parallel to the 45 degree line. But for the other three subperiods there are substantial deviations from unit slopes. The inflation on money growth scatter is steeper than 45 degrees during 1900–1928, flatter during 1929–1954, and even negative during

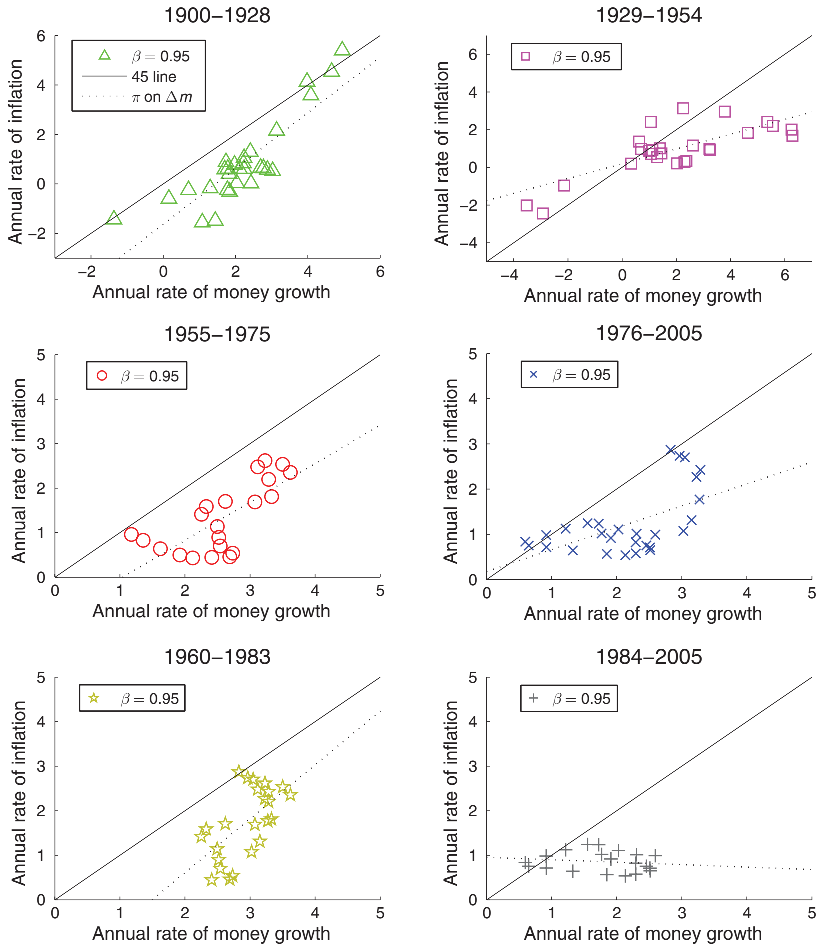

FIGURE 3. SCATTER PLOTS OF FILTERED INFLATION AND FILTERED MONEY GROWTH USING LUCAS'S FORMULA

Note: Results are reported for the second quarter of each year.

1984–2005; while the interest on money growth scatter is flatter than the 45 degree line during 1900–1928 and negatively sloped during 1929–1954 and 1984–2005.[6]

C. *Regressions on Filtered Data*

Table 1 reports regression coefficients of inflation and the nominal interest rate on money growth for filtered data using different values of β ranging from 0.95 to 0.

As in Lucas's graphs, the entries of Table 1 reveal that the closer is β to one (and therefore the smoother are the series) the larger are the regression slopes of filtered data. However, with a few exceptions concentrated in the 1955–75 and 1960–1983 periods, most estimates are significantly different from one and they span values between -0.03 and 1.13 for money growth and inflation at $\beta = 0.95$, and -0.08 and 0.75 for money growth and the nominal interest rate. In the Appendix, we show

[6] We obtain similar results using the band-pass filter proposed by Lawrence Christiano and Terry Fitzgerald (2003), and also employed by Luca Benati (2005), on frequency above eight or 20 years.

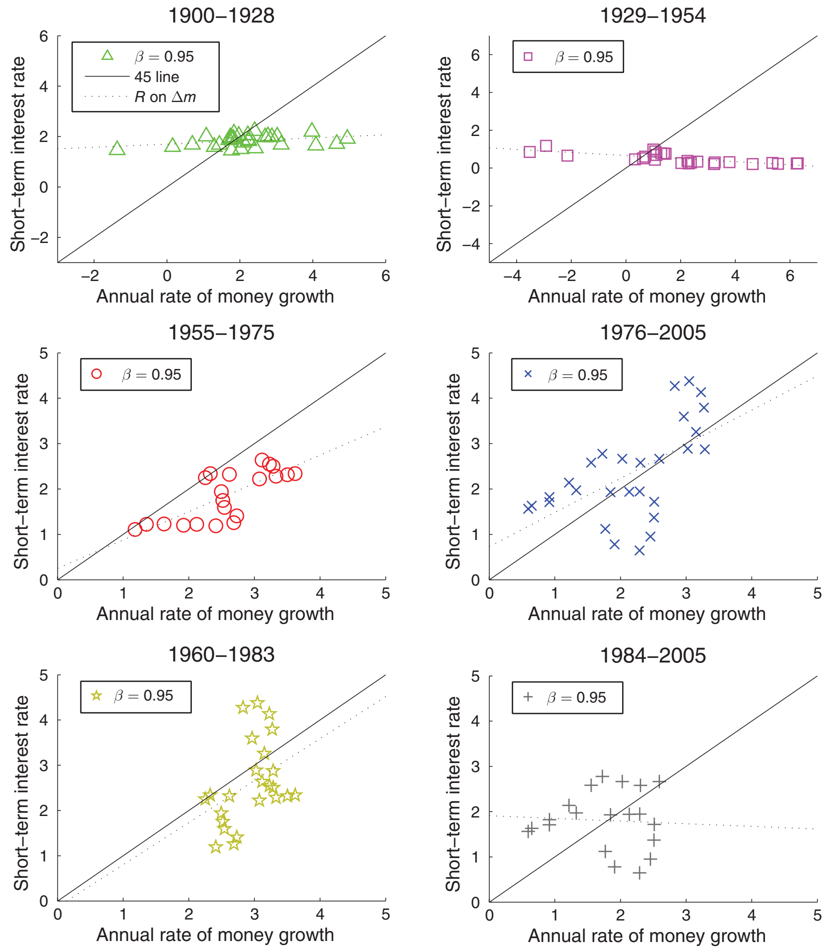

Figure 4. Scatter Plots of Filtered Short-Term Interest Rate and Filtered Money Growth Using Lucas's Formula

Note: Results are reported for the second quarter of each year.

that the message from Table 1 is not altered by using different measures of inflation, money, or short-term interest rate.

D. *Evidence from a Time-Varying VAR*

In this section, we use a time-varying VAR with stochastic volatility to construct "temporary" estimates of $\tilde{h}(0)$ that vary over time.[7] There are at least three good interconnected reasons to allow for such time variation. First, the dynamics of money growth, inflation, nominal interest rate, and output growth have exhibited substantial instabilities over a century that witnessed two world wars, a Great Depression,

[7] The description of the statistical model is presented in Sargent and Surico (2008), who followed Timothy Cogley and Sargent (2005) and Giorgio Primiceri (2005), and therefore it will not be repeated here. The full sample is 1875:1–2007:IV. A training sample of 25 years is used to calibrate the priors. Results are based on 500,000 Gibbs sampling repetitions.

Table 1—Coefficients of the Regressions on Filtered Data, 1900–2005

Data–m: M2; p: GNP/GDP deflator; R: 6-month commercial paper rate

	π on Δm				R on Δm			
β	0.95	0.8	0.5	0	0.95	0.8	0.5	0
1900–2005	0.58	0.57	0.56	0.54	0.07	0.05	0.02	0.01
1900–1928	1.13	1.18	1.21	1.15	0.06	0.04	0.00	−0.01
1929–1954	0.39	0.39	0.37	0.34	−0.08	−0.07	−0.06	−0.06
1955–1975	**0.86**	0.69	0.36	0.22	0.62	0.45	0.13	0.00
1976–2005	0.48	0.45	0.38	0.32	0.75	0.74	0.66	0.56
1960–2005	0.59	0.52	0.36	0.27	0.52	0.45	0.28	0.18
1960–1983	**1.01**	0.53	0.06	−0.04	**0.70**	0.26	−0.18	−0.25
1984–2005	−0.03	−0.04	−0.05	−0.05	0.06	0.06	0.04	0.00

Note: Numbers in bold are not statistically different from one at 10 percent significance level using heteroskedasticity and autocorrelation consistent standard errors.

the great inflation, and then a great moderation. Second, our long sample arguably transcends several monetary regimes, starting with a gold standard and ending with the fiat standard supported by a dual mandate to promote high employment and stable prices that succeeded Bretton Woods. Third, the results in the previous section are based on a subsample selection that, while consistent with Lucas (1980) and Boschen and Otrok (1994), is admittedly arbitrary.

In Figure 5, we report as red solid lines the central 68 percent posterior bands of the following object constructed from our time-varying VAR:

$$\tilde{h}_{yx,t|T}(0) = \frac{S_{yx,t|T}(0)}{S_{x,t|T}(0)}, \tag{8}$$

namely, the temporary cross-spectrum divided by the temporary spectrum at t, formed from the smoothed estimates of the time-varying VAR conditioned on the dataset $1,\ldots,T$. We compute the temporary spectral objects by applying formulas (7) and (3) and to the (t,T) versions of A, B, C, D.

We view equation (8) as a local-to-date t approximation of equation (3). Ideally, when extracting the low-frequency relationships, we should also account for the fact that the parameters drift going forward from date t. But this is computationally challenging because it requires integrating a high-dimensional predictive density across all possible paths of future parameters. Adhering to a practice in the learning literature (referred to as "anticipated-utility" by David Kreps 1998), we instead update the elements of θ_t, H_t, and A_t period-by-period and then treat the updated values as if they would remain constant going forward in time.

For comparison, we also report as blue dotted (solid) lines the 68 percent posterior bands (median values) based on the estimates for the full-sample from a fixed-coefficient VAR in money growth, inflation, the nominal interest rate, and output growth, whose details can be found in Sargent and Surico (2008).

The medians of the distributions of the $\tilde{h}(0)$s display substantial time variation. The posteriors reveal substantial uncertainty about the $\tilde{h}(0)$s, however, and in some episodes like the 1970s, $\tilde{h}(0)$ values of zero and one are simultaneously inside the posterior bands in both panels. The most recent 20 years as well as the 1940s are characterized by the lowest values of the median estimates and the smallest uncertainty. The 1970s, in contrast, are associated with the highest values and the largest uncertainty.

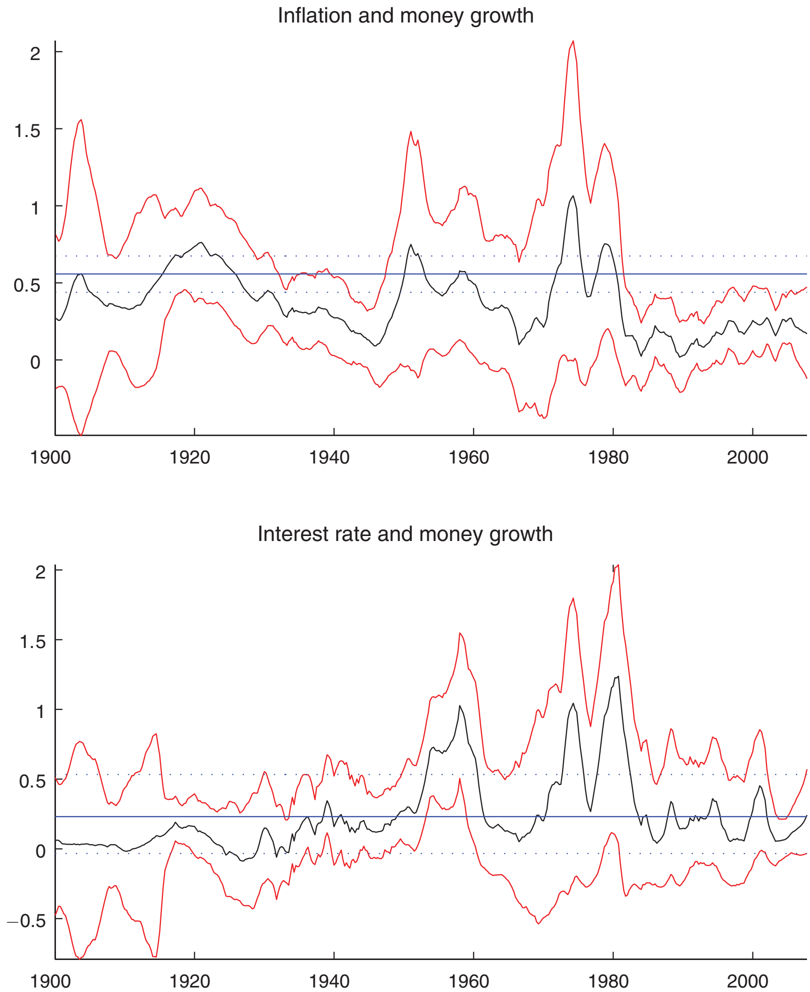

FIGURE 5. MEDIAN AND 68 PERCENT CENTRAL POSTERIOR BANDS FOR $\tilde{h}_{\pi,\Delta m}(0)$ AND $\tilde{h}_{R,\Delta m}(0)$ BASED ON A FIXED-COEFFICIENT VAR OVER THE FULL SAMPLES AND A VAR WITH TIME-VARYING COEFFICIENT AND STOCHASTIC VOLATILITY

The median estimates of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ based on the fixed coefficient multivariate BVAR for the *full sample* are 0.55 and 0.25 respectively.

As for the unit coefficients associated with the quantity theory of money, the value of one is outside the posterior bands for most of the sample, with exceptions typically concentrated in the 1970s. A comparison over different subperiods between the results based on the time-varying VAR and the straight lines from the fixed-coefficient VAR reveals that the two models can yield very different results.

III. Interpreting the Observed Instabilities with a DSGE Model

To investigate the extent to which changes in monetary policy can account for observed changes in our $\tilde{h}(0)$ statistics between nominal variables, we proceed in three steps. First, we describe a version of what is currently a popular model for monetary policy analysis, a model that under the appropriate monetary policies is well within the class of models capable of illustrating Lucas's two quantity theory propositions. Second, we estimate the parameters of the model over a post–World War II subsample that, arguably, was characterized by a homogenous policy regime. Third, we lock all except the monetary policy parameters at their estimated values from the post–World War II subsample and then vary the coefficients describing the policy response to inflation and output over a broad range of values.[8] Then, for each pair of policy coefficients, we compute the implied $\tilde{h}(0)$ statistics. By proceeding in this way, we aim to assess how well, within our estimated DSGE model, changes in monetary policy alone can account for the changes observed in the low frequency relationships between money growth and inflation, and money growth and the nominal interest rate.[9]

A. *A Model for Monetary Policy Analysis*

In this section, we lay out the log-linearized version of a model with sticky price, price indexation, habit formation, and unit root technology shocks derived by Peter Ireland (2004). While our results are not sensitive to this particular choice, it makes sense to frame our analysis within a model that has become popular in some policy and academic circles.[10]

The structure of the economy is:

$$\pi_t = \theta(1 - \alpha_\pi)E_t\pi_{t+1} + \theta\alpha_\pi\pi_{t-1} + \kappa x_t - \frac{1}{\tau}e_t, \tag{9}$$

$$x_t = (1 - \alpha_x)E_t x_{t+1} + \alpha_x x_{t-1} - \sigma(R_t - E_t\pi_{t+1}) + \sigma(1 - \xi)(1 - \rho_a)a_t, \tag{10}$$

$$\Delta m_t = \pi_t + z_t + \frac{1}{\sigma\gamma}\Delta x_t - \frac{1}{\gamma}\Delta R_t + \frac{1}{\gamma}(\Delta\chi_t - \Delta a_t), \tag{11}$$

$$\tilde{y}_t = x_t + \xi a_t, \quad \Delta y_t = \tilde{y}_t - \tilde{y}_{t-1} + z_t, \tag{12}$$

where π_t, x_t, Δm_t, and R_t are inflation, the output gap, nominal money growth, and the short-term interest rate, respectively. The level of detrended output is $\tilde{y}_t$, and Δy_t refers to output growth. The rate of technological progress is z_t. Equation (9) is an example of a new Keynesian Phillips curve, while (10) is called the new Keynesian

[8] Friedman and Schwartz (1963) documented significant changes in the monetary operations of the US government and, after 1914, the Fed over the first half of our sample period. More recently, Ibrahim Chowdhury and Andreas Schabert (2008) have shown that the systematic component of a Fed money supply rule shifted significantly during the early 1980s. As for interest rate rules, Richard Clarida, Jordi Galí, and Mark Gertler (2000), Thomas Lubik and Frank Schorfheide (2004), and Jesus Fernandez-Villaverde, Pablo Guerrón, and Juan F. Rubio-Ramirez (2009), among many others, have argued that the new policy regime established by Paul Volcker during the first years of his mandate as Fed chairman represented an unprecedented break in the conduct of US monetary policy.

[9] Other factors such as financial innovation may have also contributed to the instabilities documented in Section II. Investigating the role of diminishing financial frictions, however, would require a different model relative to the current workhorse, and it is beyond the scope of this paper.

[10] Sargent and Surico (2008) show that changes in monetary policy induce significant changes in the low-frequency relationships between nominal variables also in a neoclassical model à la Lucas (1975) that was also featured by Whiteman (1984).

IS curve. Equation (11) is a money demand equation of the type derived by Bennett McCallum and Edward Nelson (1999) and Ireland (2003).

The discount factor is θ, the parameter α_π is price setters' extent of indexation to past inflation, and α_x captures the extent of habit formation. The coefficients κ and σ are the slope of the Phillips curve and the elasticity of intertemporal substitution in consumption. The price adjustment cost parameter in Julio Rotemberg's (1982) quadratic function is τ, while ξ represents the inverse of the Frisch labor elasticity. The inverse of the interest elasticity of money demand is γ.

The economy is exposed to four nonpolicy disturbances, namely, a markup shock e_t, a demand shock a_t, a money demand shock χ_t, and a technology shock Z_t that evolve as

$$
\begin{aligned}
e_t &= \rho_e e_{t-1} + \varepsilon_{et}, \text{ with } \varepsilon_{et} \sim N(0, \sigma_e^2), \\
a_t &= \rho_a a_{t-1} + \varepsilon_{at}, \text{ with } \varepsilon_{at} \sim N(0, \sigma_a^2), \\
\chi_t &= \rho_\chi \chi_{t-1} + \varepsilon_{\chi t}, \text{ with } \varepsilon_{\chi t} \sim N(0, \sigma_\chi^2), \\
\Delta \ln(Z_t) \equiv z_t &= \varepsilon_{zt}, \text{ with } \varepsilon_{zt} \sim N(0, \sigma_z^2).
\end{aligned}
$$

All variables are expressed in log deviations from their steady state values.

We consider two types of monetary policy rules, each of which, depending on parameter values, allows extensive feedback from endogenous variables to money growth.[11] The first is a money growth rule according to which the central bank adjusts smoothly the growth rate of money in response to movements in inflation and the output gap.

$$
(13) \quad \Delta m_t = \rho_m \Delta m_{t-1} + (1 - \rho_m)(\phi_\pi \pi_t + \phi_x x_t) + \varepsilon_{mt}, \varepsilon_{mt} \sim N(0, \sigma_m^2).
$$

The second is a Taylor rule according to which the short-term nominal interest rate is adjusted smoothly in response to movements in inflation and the output gap.

$$
(14) \quad R_t = \rho_r R_{t-1} + (1 - \rho_r)(\psi_\pi \pi_t + \psi_x x_t) + \varepsilon_{Rt}, \quad \varepsilon_{rt} \sim N(0, \sigma_R^2).
$$

B. *Estimation*

We use Bayesian methods to characterize the posterior distribution of the parameters of the model (see Sungbae An and Schorfheide, 2007, for how to use Bayesian methods to estimate linearized DSGE models). As in the VAR, we specify the vector of observable variables as $[\Delta m_t, \pi_t, R_t, \Delta y_t]$. The estimation sample is 1960:I–1983:IV, and the policy rule is the money supply in (13). The reason behind these choices is twofold. First, the pre-1984 period is characterized by far larger variation in money growth, inflation, output, and short-term interest rate than the

[11] Whiteman (1984) assumed that money growth is econometrically exogenous in the sense of Christopher A. Sims (1972). For us, depending on monetary policy rule parameter values, other variables can Granger cause money growth rates (see Clive W. J. Granger 1969, and Sims 1972).

post-1984 period. This is attractive because it implies that the data over this period are likely to be more informative. Second, estimating a Taylor rule over the pre-1984 sample is complicated by the possibility of multiple equilibria, which brings up contentious issues of equilibrium selection in the indeterminacy regime. A money supply rule, in contrast, makes the economy less prone to multiple equilibria while perhaps providing a better representation of monetary policy during the pre-1984 regime, including Volcker's experiment of nonborrowed reserve targeting, which is typically excluded from the estimation of DSGE models with a Taylor rule (see, for instance, Frank Smets and Rafael Wouters 2007, and Cogley, Primiceri, and Sargent 2010).

In Table 2, we report our priors, which are relatively disperse around values from previous studies. The priors of the model parameters imply very disperse priors for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ in the last two rows. Our priors on the policy coefficients ϕ_π and ϕ_x on inflation and output, respectively, in the money supply rule are relatively uninformative and centered at zero. The priors on the slope of the Phillips curve and on the interest rate semi-elasticity of aggregate demand imply a larger impact of output on inflation than of the interest rate on output. The prior on the discount factor is tight, while those on the coefficients governing price adjustment, Frisch elasticity, and interest rate semi-elasticity of money demand are quite disperse reflecting the difficulties of estimating these parameters on aggregate data. The processes of the shocks all have same persistence and variance a priori.

The last three columns of Table 2 report the mean, the fifth, and ninety-fifth percentiles of the posterior distributions. In line with the results from earlier contributions, the Phillips curve has a larger backward-looking component and a flatter slope than the IS curve. The other coefficients describing the structure of the economy are not precisely estimated, and their distributions appear to cover the range of available estimates using macro data. The policy parameters $(\phi_\pi, \phi_x) \approx (0.23, -0.2)$ and $\rho_m \approx 0.74$ indicate that during our sample period the Fed was moving money growth smoothly with relatively little weight on inflation and the output gap, thereby putting variations into the money growth that allowed the two quantity theory propositions to express themselves in the way formulated by Lucas. Finally, the demand shock is associated with the most persistent process, whereas technology shocks have the largest variance. The posteriors of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ are considerably tighter than the priors, and both shifted to the right to values consistent with the estimates in Table 1 and Figure 5.

C. *Assessing Variation in Monetary Policy*

In this subsection, we execute step 3 of our experiment: we lock all of the structural parameters of our DSGE model, except the policy coefficients (ϕ_π, ϕ_x), at the posterior means reported in Table 2 and let (ϕ_π, ϕ_x) vary in the intervals $[-3, 1]$ and $[-1, 0]$, respectively. For each pair of policy coefficients, we then compute the low-frequency relationships between money growth, inflation, and the nominal interest rate. The results from this exercise are displayed in Figure 6, where the values of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ are displayed as contour plots that vary with the conduct of monetary policy. We also report as dark scatter plots the joint posterior distribution of the policy parameters. Conditional on the posterior means of Table 2, Figure 6

Table 2—Prior Densities and Posterior Estimates

	Prior				Posterior		
Coefficient	Density	Domain	Mean	SD	Mean	[5th ;	95th]
θ	theta	$[0, 1]$	0.99	0.005	0.9901	[0.9830 ;	0.9976]
α_π	beta	$[0, 1]$	0.5	0.2	0.8815	[0.7902 ;	0.9783]
κ	gamma	$\mathbb{R}^+$	0.3	0.1	0.0324	[0.0143 ;	0.0492]
τ	gamma	$\mathbb{R}^+$	4	1	3.5126	[1.9869 ;	4.9799]
α_x	beta	$[0, 1]$	0.5	0.2	0.4775	[0.4547 ;	0.5000]
σ	gamma	$\mathbb{R}^+$	0.1	0.05	0.0997	[0.0643 ;	0.1355]
ξ	gamma	$\mathbb{R}^+$	2	1	3.0319	[1.2482 ;	4.7369]
γ	gamma	$\mathbb{R}^+$	4	1	3.8128	[3.0866 ;	4.5332]
ϕ_π	normal	$\mathbb{R}$	0	0.5	0.2312	[−0.0504 ;	0.5157]
ϕ_x	normal	$\mathbb{R}$	0	0.5	−0.1971	[−0.3092 ;	−0.0814]
ρ_m	beta	$[0, 1]$	0.5	0.05	0.7428	[0.7064 ;	0.7902]
ρ_e	beta	$[0, 1]$	0.5	0.1	0.5645	[0.4472 ;	0.6844]
ρ_a	beta	$[0, 1]$	0.5	0.1	0.9241	[0.9006 ;	0.9528]
ρ_χ	beta	$[0, 1]$	0.5	0.1	0.5024	[0.3363 ;	0.6649]
σ_e	inverse gamma	$\mathbb{R}^+$	0.3	1	1.0922	[0.5608 ;	1.5936]
σ_a	inverse gamma	$\mathbb{R}^+$	0.3	1	0.7226	[0.3230 ;	1.1067]
σ_χ	inverse gamma	$\mathbb{R}^+$	0.3	1	0.2388	[0.0721 ;	0.4229]
σ_z	inverse gamma	$\mathbb{R}^+$	0.3	1	1.5845	[1.2941 ;	1.8764]
σ_m	inverse gamma	$\mathbb{R}^+$	0.3	1	1.1457	[0.9866 ;	1.2978]
implied $\tilde{h}_{\pi,\Delta m}(0)$		$\mathbb{R}$	0.65	0.40	1.0068	[0.9909 ;	1.0261]
implied $\tilde{h}_{R,\Delta m}(0)$		$\mathbb{R}$	0.31	0.64	0.8163	[0.6183 ;	0.9764]

Notes: Sample—1960:I–1983:IV. Results are based on 500,000 draws from the posterior using the Metropolis-Hastings algorithm and a fraction of accepted draws of 29 percent.

therefore illustrates the mapping from pairs of policy coefficients to population values of the $\tilde{h}(0)$s implied by our DSGE model. For instance, the posterior means of the policy coefficients in Table 2 are associated with values of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ that lie just above the 1.0 and 0.8 contours, respectively.

Inspection of Figure 6 reveals three main findings. First, the low-frequency relationships between money growth and inflation, and money growth and the nominal interest rate, are not policy invariant: changes in monetary policy can account, on their own, for movements in $\tilde{h}_{\pi,\Delta m}(0)$ $\left(\tilde{h}_{R,\Delta m}(0)\right)$ between 0.1 (0) and 1 (1). Second, the joint posterior distribution of the policy parameters implies values of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ consistent with the estimates from both the regression slopes of filtered data and the time-varying VAR. Third, a monetary policy shift towards a significantly more aggressive anti-inflationary stance, as exemplified by values of ϕ_π smaller than -1.5 and values of ϕ_x close to zero, can generate values for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ which are consistent with the estimates in Table 1 and Figure 5.

To make the last point more transparent, we have reported as lighter areas in Figure 6 the subspace of ϕ_π and ϕ_x values for which $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ are simultaneously below 0.2. For instance, the pair of policy coefficients $\phi_\pi = -2.5$ and $\phi_x = 0$, which are consistent with the behavior of a central bank that cares about bringing inflation gradually back to target, imply population values of $\tilde{h}_{\pi,\Delta m}(0) = 0.1$ and $\tilde{h}_{R,\Delta m}(0) = 0$ in the DSGE model. These values are not statistically different from our estimates in Table 1, and they fall well within the posterior bands of the post-1984 estimates in Figure 5.

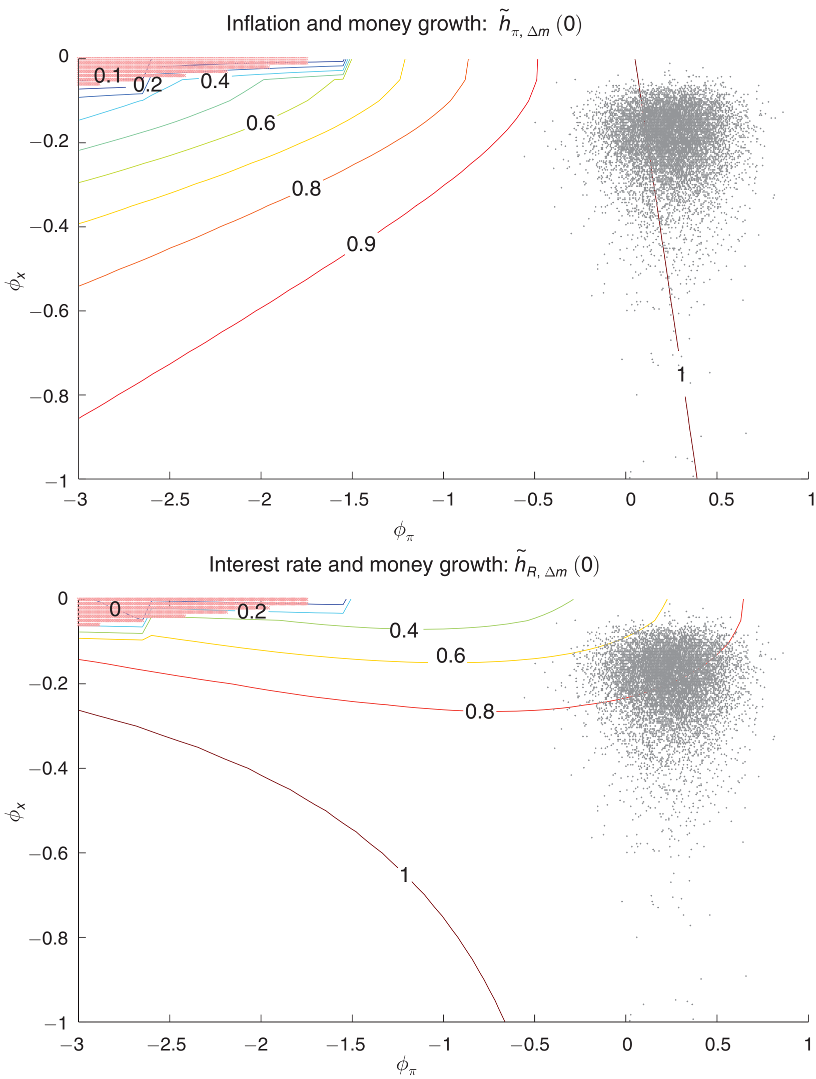

FIGURE 6. SUMS OF WEIGHTS $\tilde{h}(0)$ IN THE ESTIMATED NEW KEYNESIAN MODEL UNDER A MONEY SUPPLY RULE

Notes: The darker areas denote the joint posterior distribution of ϕ_π and ϕ_x; the lighter areas denote values of ϕ_π and ϕ_x for which both $\tilde{h}_{\pi,\Delta m}(0) < 0.2$ *and* $\tilde{h}_{R,\Delta m}(0) < 0.2$.

A similar exercise but for a Taylor rule is reported in Figure 7. Whenever equilibrium indeterminacy arises, we use the orthogonality solution in Lubik and Schorfheide (2004) and set the variance of sunspot shocks to their estimated value of 0.2. The main findings from this exercise can be summarized as follows. That the $\tilde{h}(0)$s depend on the parameters of the policy rule continues to come through when we use a Taylor rule as a description of monetary policy. But the model is less able to replicate the estimates of $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ because there is no region of the policy parameter space that we considered for which the implied values of the low-frequency relationships are simultaneously either within the bands of the estimates in Table 2 or below 0.2. In the next section, we will explore the extent to

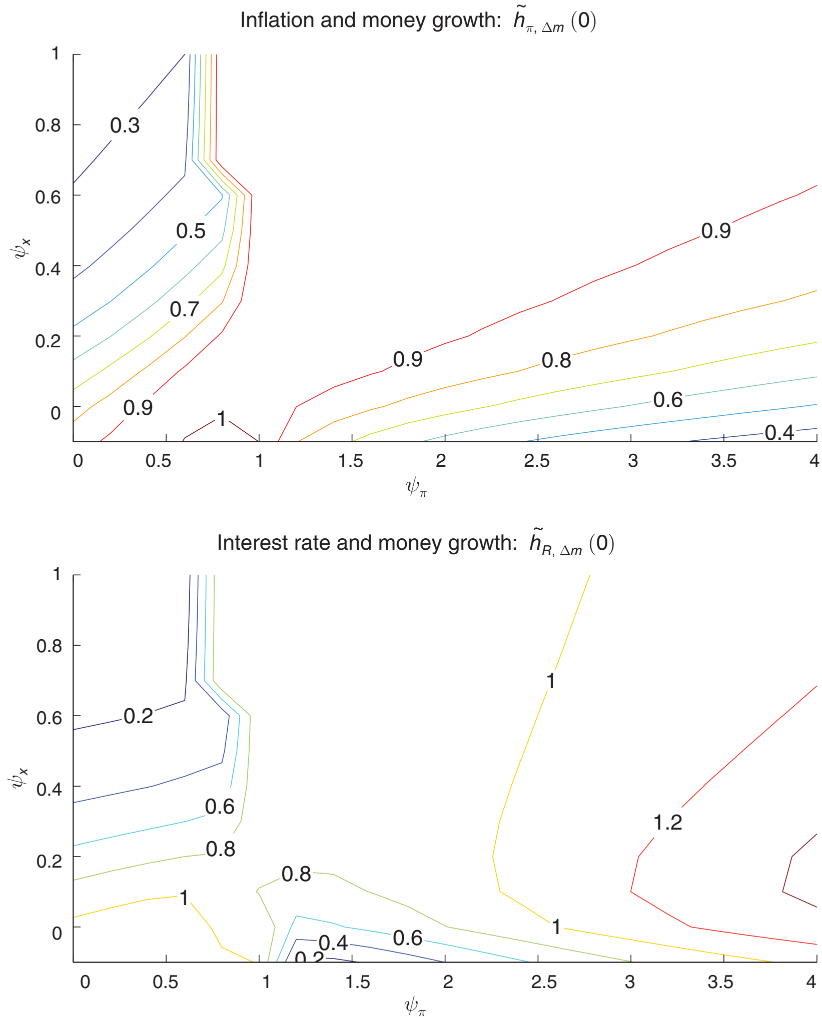

Figure 7. Sums of Weights $\tilde{h}(0)$ in New Keynesian Model under a Taylor Rule

which these outcomes can (or cannot) be improved by changing the values of other model parameters.

D. *Sensitivity Analysis*

Several contributions have found that the variance of supply shocks declined remarkably when moving from the pre-1984 to the post-1984 sample (see for instance Lubik and Schorfheide 2004, and Ireland 2004). In this section, we assess the robustness of the $\tilde{h}(0)$s to a fall in the variance of supply shocks.

In Figure 8, we repeat the same calculations as in figures 6 and 7 with one exception: the standard deviation of supply shocks, σ_e, is now set to 0.273, a value four times smaller than the posterior mean in Table 2. Two main results emerge from this sensitivity analysis. First, a change in the variance of supply shocks of the magnitude considered in Figure 8 is insufficient to make $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ policy

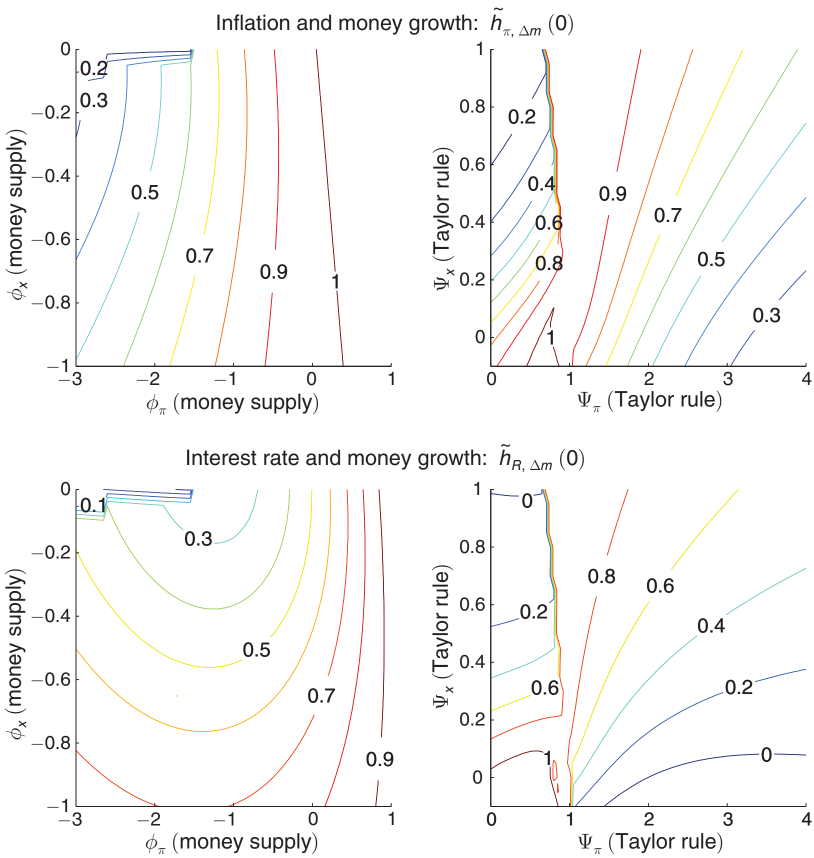

FIGURE 8. SUMS OF WEIGHTS $\tilde{h}(0)$ IN THE ESTIMATED NEW KEYNESIAN MODEL UNDER MONEY SUPPLY AND TAYLOR RULES

Note: The standard deviation of supply shocks, σ_e, is set to 0.273.

invariant. Second, under the interest rate rule, a more anti-inflationary policy stance is now associated with considerably lower values for $\tilde{h}_{\pi,\Delta m}(0)$ and $\tilde{h}_{R,\Delta m}(0)$ than in Figure 7. It should be noted, however, that very different pairs of ψ_π and ψ_x in the top-left corner of each panel generate values of the $\tilde{h}(0)$s which are similar to the values associated with pairs of policy coefficients in the opposite corner.[12]

In summary, our results indicate that by reacting sufficiently aggressively to incipient inflationary pressures, a monetary rule can eradicate the two unit regression coefficients that manifest Lucas's quantity theory propositions. But if a monetary rule unleashes persistent and seemingly exogenous movements in money growth, as implied, for example, by $\phi_\pi = \phi_x = 0$ or $\psi_\pi = \psi_x = 0$, then Lucas's two illustrations of the quantity theory will come back.

[12] Changes in the variances of the other shocks, in contrast, generate only little variation in the $\tilde{h}(0)$s. Results are available upon request.

IV. Concluding Remarks

For reasons that Lucas (1972), Sargent (1971), and Robert King and Watson (1994, 1997) described in the context of econometric tests of the natural unemployment rate hypothesis and that Whiteman (1984) analyzed in the context of the quantity theory of money, low-frequency properties of two-sided infinite projections depend on government policies. This is why Lucas (1980) qualified his post–World War II illustrations of two quantity theory propositions with an important caveat about when the propositions will "obtain in reality" and when they will "break down." In this paper, we have documented periods in which the two quantity theory propositions have broken down and, within an estimated DSGE model, have also identified alterations in monetary policies that can account for those breakdowns.

As citizens, we prefer times when the propositions *have* broken down because Lucas's unit slopes can be expected to emerge when a monetary authority allows persistent movements in money growth by responding too weakly to inflationary pressures.

Appendix: Robustness of the Empirical Results

Table A—Coefficients of the Regressions on Filtered Data, Robustness

	π on Δm				R on Δm			
Quarterly data–m: M2; R: 3-month TB rate								
β					0.95	0.8	0.5	0
1900–2005					0.07	0.02	0.00	−0.01
1900–28					0.03	0.04	0.02	0.01
1929–54					−0.11	−0.11	−0.11	−0.11
1955–75					0.85	0.71	0.44	0.32
1976–05					0.68	0.67	0.63	0.53
Quarterly data–m: M1; p: CPI; R: 3m TB rate—(as in Lucas 1980)								
β	0.95	0.8	0.5	0	0.95	0.8	0.5	0
1955–2005	0.45	0.36	0.21	0.13	0.48	0.37	0.21	0.13
1955–75	1.39	1.22	0.74	0.46	0.90	0.80	0.51	0.35
1976–05	0.30	0.24	0.13	0.08	0.39	0.28	0.14	0.07
Quarterly data–m: M1; R: 6m commercial paper rate rate								
β					0.95	0.8	0.5	0
1955–2005					0.51	0.39	0.22	0.15
1955–75					**0.99**	0.88	0.55	0.37
1976–05					0.41	0.30	0.15	0.10
Annual data–m: M1; p : GNP/GDP deflator; R: 3m TB rate–$k = 2$								
β	0.95	0.8	0.5	0	0.95	0.8	0.5	0
1900–2005	0.52	0.52	0.51	0.52	0.04	0.04	0.03	0.00
1900–54	0.52	0.52	0.53	0.57	−0.06	−0.06	−0.05	−0.04
1955–05	0.48	0.46	0.37	0.14	**0.80**	0.77	0.62	0.22
Annual data–m: M1; R: 6m commercial paper rate–$k = 2$								
β					0.95	0.8	0.5	0
1900–2005					−0.01	−0.01	−0.02	−0.04
1900–54					−0.11	−0.11	−0.10	−0.07
1955–05					**0.81**	0.77	0.63	0.21

Notes: Numbers in bold are not statistically different from 1 at 10% significance level, HAC covariance matrix. The definition of M1 was broadened in 1980 to include nonbank checkable deposits. Annual data are from Ireland (2009).

REFERENCES

An, Sungbae, and Frank Schorfheide. 2007. "Bayesian Analysis of DSGE Models." *Econometric Reviews*, 26(2–4): 113–72.

Balke, Nathan S., and Robert J. Gordon. 1986. "Appendix B: Historical Data." In *The American Business Cycle: Continuity and Change.* National Bureau of Economic Research Conference on Research in Business Cycles, ed. Robert J. Gordon, 781–850. Chicago: University of Chicago Press.

Benati, Luca. 2005. "Long-Run Evidence on Money Growth and Inflation." *Bank of England Quarterly Bulletin*, 45(3): 349–55.

Boschen, John F., and Christopher M. Otrok. 1994. "Long-Run Neutrality and Superneutrality in an ARIMA Framework: Comment." *American Economic Review*, 84(5): 1470–73.

Chowdhury, Ibrahim, and Andreas Schabert. 2008. "Federal Reserve Policy Viewed through a Money Supply Lens." *Journal of Monetary Economics*, 55(4): 825–34.

Christiano, Lawrence, and Terry Fitzgerald. 2003. "The Band Pass Filter." *International Economic Review*, 44(2): 435–65.

Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*, 115(1): 147–80.

Cogley, Timothy, and Thomas J. Sargent. 2005. "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US." *Review of Economic Dynamics*, 8(2): 262–302.

Cogley, Timothy, Giorgio Primiceri, and Thomas J. Sargent. 2010. "Inflation-Gap Persistence in the US." *American Economic Journal: Macroeconomics*, 2(1): 43–69.

Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez. 2009. "Fortune or Virtue: Time-Variant Volatilities Versus Parameter Drifting in U.S. Data." Unpublished.

Fisher, Mark E., and John J. Seater. 1993. "Long-Run Neutrality and Superneutrality in an ARIMA Framework." *American Economic Review*, 83(3): 402–15.

Friedman, Milton, and Anna J. Schwartz. 1963. *A Monetary History of the United States, 1867–1960.* Princeton, NJ: Princeton University Press.

Granger, Clive W. J. 1969. "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods." *Econometrica*, 37(3): 424–38.

Ireland, Peter. 2003. "Endogenous Money or Sticky Prices?" *Journal of Monetary Economics*, 50(8): 1623–48.

Ireland, Peter. 2004. "Technology Shocks in the New Keynesian Model." *Review of Economics and Statistics*, 86(4): 923–36.

Ireland, Peter. 2009. "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review*, 99(3): 1040–52.

King, Robert, and Mark W. Watson. 1994. "The Post-War U.S. Phillips Curve: A Revisionist Econometric History." *Carnegie-Rochester Conference Series on Public Policy*, 41(12): 157–219.

King, Robert, and Mark W. Watson. 1997. "Testing Long-Run Neutrality." *Federal Reserve Bank of Richmond Economic Quarterly*, 83(3): 69–101.

Kreps, David. 1998. "Anticipated Utility and Dynamic Choice." In *Frontiers of Research in Economic Theory: The Nancy L Schwartz Memorial Lectures, 1983–1997*, ed. Donald P. Jacobs, Ehud Kalai, and Morton I. Kamien, 242–74. New York: Cambridge University Press.

Lubik, Thomas, and Frank Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review*, 94(1): 190–217.

Lucas, Robert E. Jr. 1972. "Econometric Testing of the Natural Rate Hypothesis." In *Econometrics of Price Determination: Conference, October 30–31, 1970, Washington, DC,* ed. Otto Eckstein, 50–59. Washington, DC: Board of Governors of the Federal Reserve System.

Lucas, Robert E., Jr. 1975. "An Equilibrium Model of the Business Cycle." *Journal of Political Economy*, 83(6): 1113–44.

Lucas, Robert E., Jr. 1980. "Two Illustrations of the Quantity Theory of Money." *American Economic Review*, 70(5): 1005–14.

McCallum, Bennett, and Edward Nelson. 1999. "Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model." In *Monetary Policy Rules.* National Bureau of Economic Research Business Cycles Series, Vol. 31, ed. John B. Taylor, 15–45. Chicago: University of Chicago Press.

Phillips, Peter C. B. 1991. "Spectral Regression for Cointegrated Time Series." In *Nonparametric and Semiparametric Methods in Econometrics and Statistics: Proceedings of the Fifth International Symposium in Economic Theory and Econometrics,* ed. William A. Barnett, James Powell, and George Tauchen, 413–36. New York: Cambridge University Press.

Primiceri, Giorgio. 2005. "Time Varying Structural Vector Autoregressions and Monetary Policy." *Review of Economic Studies*, 72(3): 821–52.

Rotemberg, Julio. 1982. "Sticky Prices in the United States." *Journal of Political Economy*, 90(6): 1187–1211.

Sargent, Thomas J. 1971. "A Note on the 'Accelerationist' Controversy." *Journal of Money, Credit, and Banking*, 3(3): 721–25.

Sargent, Thomas J., and Paolo Surico. 2008. "Monetary Policies and Low-Frequency Manifestations of the Quantity Theory." Bank of England External Monetary Policy Committee Unit Discussion Paper 26.

Sims, Christopher A. 1972. "Money, Income, and Causality." *American Economic Review*, 62(4): 540–52.

Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586–606.

Stock, James H., and Mark W. Watson. 1993. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems." *Econometrica*, 61(4): 783–820.

Whiteman, Charles H. 1984. "Lucas on the Quantity Theory: Hypothesis Testing without Theory." *American Economic Review*, 74(4): 742–49.